From Local to Global Competition*

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ABSTRACT

We introduce a framework that has known models of oligopolistic competition with differentiated products (the circle, the logit, and the CES) as limit cases. This integrative approach incorporates both localized and global competition, as well as price-sensitive individual demands. It is used to explain the impact of major changes over the last two centuries: reductions in transport costs, increased taste for variety, population growth, and use of technologies with greater returns to scale. We work out the properties of an extended Chamberlinian model with applications both in Industrial Organization and Economic Geography.

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1. INTRODUCTION

The changes in the structure of retailing in towns, and industrial structure in general, have been enormous over the last two centuries. Much of what has happened can be ascribed to changes in economic fundamentals, including technological changes, transport improvements, population growth, and changes in tastes. When economists want to think about explaining the changes in industrial structure and retailing, it is natural to turn to oligopoly theory. Within the existing theory, there are two basic choices for models of oligopoly with price competition. These are the spatial model such as the circle model developed by Vickrey (1964) and elaborated by Salop (1979); and the (aspatial) representative consumer model such as the CES model used by Spence (1976) and Dixit and Stiglitz (1977), which is the workhorse in much of the work by Krugman in economic geography and trade (for example, Krugman, 1995; see also Grossman and Helpman, 1991).

These approaches are very different. Spatial models such as the circle model or the Hotelling (1929) model are useful because they admit an explicit role for space and generate market demands by explicitly integrating over consumers with different tastes. But the standard location model assumes consumers buy one unit of the product, and that goods sold by firms are perfectly homogenous so that consumers go to the nearest store (in equilibrium) to buy their unit. This is a rather simplistic description of individual demand and individual travel behavior. The CES representative consumer model starts directly with market demand. It allows aggregate demand to vary with the price level, but this demand is unitary elastic. The model admits (nonspatial) product differentiation, but has the strong symmetry property that a price cut draws demand equally from all other firms. Moreover, the ratio of any two demands is independent of the prices of all the other products. Under the CES formulation, each firm competes equally with all others (competition is global): by contrast, the circle model has each firm directly competing with only its two immediate neighbors (competition is localized). In this paper we propose a method that integrates these two approaches and retains the desirable features of each. Thus we allow the structure of competition to vary between the two extremes, while the price elasticity of aggregate demand is allowed to vary between zero or unity. Aggregate demand is generated from explicit aggregation of heterogeneous individuals. The resulting framework is more than the sum of its parts insofar as there are interactions that one would not expect from just thinking about the properties of the ingredient models.

We apply the model to study the changes described in the first paragraph. For many cases the comparative static results are simple and intuitive, and we eschew formal algebraic proofs of these in favor of verbal arguments that follow from the model. This also highlights the robustness of the results, and stresses properties common to models of oligopolistic competition. Other results are more intricate and depend on parameter values. For example, an increase in consumer taste for variety (or choosiness) can reduce the market equilibrium price if taste for variety is low, but increases it otherwise. The latter result is standard in models of product differentiation and stems from greater market power. The former result is an explicitly spatial phenomenon arising from the joint consideration of taste for variety along with spatial differentiation. The intuition is that a higher taste for variety breaks down the localization of competition in the spatial market and brings firms into market contact with other firms beyond their nearest neighbors. The consequent higher degree of contact engenders more competition for consumers farther away, and thence lower prices.

Another variable of interest, and which is special to our setting, is the degree of *effective* variety available to consumers. Large effective variety means that many differentiated products are easily accessible. The degree effective variety also provides an index of the degree of globalization of competition. An index close to zero means consumers mainly buy from the closest firm; an index close to one means consumers shop around. In a comparison of two cities with the same population density, we show that the larger one will have a *lower* effective variety (even though it has more firms) because price competition among firms keeps their numbers from rising proportionally with size. This effect is overturned if the population density in the larger city is sufficiently large, so there can indeed be more effective choice in Geneva than in Charlottesville. The model we present is highly stylized and liberally uses symmetry assumptions. It is intended to provide a very broad sketch of major forces that determine industry structure. By emphasizing both space and taste for variety, it contributes to the so-called New Economic Geography (see Krugman, 1991). We hope that the model will form a basis for future empirical studies. Recent work on the automobile market by Berry, Levinsohn, and Pakes (1995), Feenstra and Levinsohn (1995), and Koujianou Goldberg (1995) has proved very successful in applying models that integrate the supply side with discrete choice models of product differentiation. Similar research could also greatly enhance our understanding of spatial interaction in the retail sector.

The structure of the paper is as follows. Section 2 sets out a general model of oligopolistic competition with product differentiation and Section 3 describes the structure of preferences that gives rise to the integrative model of spatial and product differentiation. Section 4 finds the equilibrium price and profit, while Section 5 treats free entry and exit. The effects of a reduction in transport costs and of an increased preference for diversity are covered in Sections 6 and 7, respectively. Sections 8 and 9 do likewise for population growth and changes in cost structures. Section 10 concludes. The reader who is interested in the economic predictions rather than the technical details of the model may wish to skip the next four sections and move directly to the comparative static sections, 6 through 9. The comparative static results are summarized in Table 1 in the conclusions.

2. PRELIMINARIES

There are n firms, and each produces a single variety of a differentiated product.[1] There is a population of consumers of mass N, and each consumer buys from only one of the n firms (but the amount bought depends on the price charged). A consumer of type j is characterized by a vector of continuously distributed match values, ϵ_{ij}, where $i = 1...n$ is the firm label. These match values are instrumental in describing the utility match between consumers and firms. The (indirect) utility for a consumer of type j conditional on buying from firm i is:

$$V_{ij} = Y_j + v(p_i) + \epsilon_{ij}, \quad i = 1...n, \tag{1}$$

where Y_j is consumer j's income and $v(p_i)$ is her consumer surplus (net of the match value) if firm i's product is bought at price p_i. Consumer j chooses the firm for which V_{ij} is largest (we assume that Y_j is large enough that income is never a binding constraint). For simplicity, let

$$v(p_i) = \frac{1 - p_i^{1-\alpha}}{1-\alpha} \tag{2}$$

so that conditional demand is (using Roy's identity)

$$x(p_i) = -v'(p_i) = p_i^{-\alpha} \tag{3}$$

[1] The one-product-per-firm assumption, while standard in most oligopoly analysis, is often empirically invalid. In the geographical context (health-food stores or antique shops), the assumption is perhaps less jarring than in the pure characteristics interpretation of the model (breakfast cereals, soap powders, or cars). Anderson, de Palma, and Thisse (1992) show how the logit limit case can be extended to an analytically tractable treatment of multiproduct firms, which provides some hope for extension of the current model.

where $\alpha = -x'(p_i)p_i/x(p_i) \in [0,1)$ is the elasticity of conditional demand.[2] When $\alpha = 0$, each consumer buys one unit from her most preferred firm, while $\alpha \to 1$ corresponds to spending a constant amount from that firm.

Assume that firm i charges p_i and all other firms charge p^*. The market share of firm i is the fraction of consumers for whom $i = \underset{k=1\cdots n}{\operatorname{argmax}}[v(p_k) + \epsilon_{kj}]$ and is denoted by $S_i(p_i, p^*)$, with $S(p^*, p^*) = 1/n$. In case of ties, consumers are shared equally among firms that they like best. For the moment, let marginal cost be zero (this assumption is relaxed in Section 9). Letting K denote fixed costs, each firm's profit is

$$\Pi_i = Np_i x(p_i) S_i(p_i, p^*) - K, \quad i = 1...n. \tag{4}$$

From the first-order condition with respect to p_i, using (3) and setting $p_i = p^*$, the candidate symmetric price equilibrium is given by

$$p^* = \left[\frac{1-\alpha}{nS'}\right]^{1/(1-\alpha)}, \tag{5}$$

where $S' > 0$ denotes $\frac{\delta S_i(p_i, p^*)}{\delta v(p_i)}$ (the market share derivative with respect to conditional consumer surplus) evaluated at $p_i = p^*$. The equilibrium profit per firm is

$$\Pi^*(n) = \frac{N(1-\alpha)}{n^2 S'} - K. \tag{6}$$

All relevant magnitudes are therefore determined by S', which therefore provides a clean summary

[2] The assumption that elasticity is constant can be relaxed, but the resulting price equilibrium expressions no longer have reduced forms.

statistic enabling the comparison of different symmetric oligopoly models including those commonly considered in the literature as well as new forms. This statistic is determined by the specification of the match values introduced in (1).

3. MATCH VALUES

We wish to capture the idea that consumers buy the product that suits them best in terms of geographical location distance and product specification. If transport costs are high and products do not differ much, a consumer will not go to the other side of town even though she has a slight preference for the product sold there. If transport costs are low and products are greatly differentiated, someone will readily travel far to get a good product match (e.g. take the subway to get sushi). In the first case competition is essentially *local*, while in the second it is essentially *global*. We assume that the total match value of a consumer with a firm is a weighted sum of the geographic match and the specific product match. Varying the relative weights on these matches generates different degrees of localization of competition.

The geographic match is generated from the circle model with linear transport costs.[3] Consumers are uniformly distributed around a circle of circumference L, so the consumer density is N/L. Products are equally spaced and located L/n apart.[4] A consumer at location z_j travels a distance $|z_j - z_i|$ if she buys from a firm at location z_i (she takes the shorter way around the circle). Letting the transport cost rate be t, the geographic matches for consumer j are $-t|z_j - z_i|$,

[3]Other specifications of the transport cost function are also possible, but linear seems the most natural. We could parameterize the transport cost function, but we would not wish to burden the analysis with further parameters that we think would have little effect. More importantly, we cannot integrate the demand functions for other transport cost specifications, so we can only get a closed form price equilibrium for linear transport costs. We should note though that quadratic transport costs would give us equilibrium existence directly from Caplin and Nalebuff (1991).

[4]Ideally, we should also prove that symmetric locations constitute an equilibrium in an extended model with endogenous locations. For the present model this is a daunting task. Simpler models have such symmetric equilibria (e.g. Economides, 1989).

$i = 1...n$.

In standard location models, each firm competes directly with only two neighbors, and markets are strictly delineated in the geographic space. When all firms charge the same price, each consumer goes to the closest firm since all products are homogeneous (apart from their locations). If though products are differentiated, a consumer might buy from a more distant firm if it better suits her tastes. Market boundaries blur, and consumers at the same geographical location may purchase from different firms. From a firm's perspective, there is then a positive probability that a consumer at any given location will buy from it. In the limit as transport costs tend to zero, this probability tends to $1/n$ if all firms charge the same price.

We wish to construct a demand system that reduces to a completely symmetric one (a la Chamberlin) at the limit $t \to 0$. We therefore suppose that the product match values for a consumer j chosen at random from the population are *i.i.d.* across products. This implies that if all products carried the same price, a new entrant would capture customers equally from all existing firms. Similarly, a price cut by one firm would draw consumers equally from all other firms. We shall further concentrate on a specific distribution of consumer tastes, the double exponential, which gives rise to the logit specification for the product selection percentages. The logit is the only form that has the additional symmetry property that the ratio of choice percentages for any two products is independent of the price - or even the existence - of any third product. This is known in the literature as the Independence of Irrelevant Alternatives (IIA) property, which implies that a new entrant or a product price reduction will take customers from other products in proportion to the original choice percentages.

Now consider how the demand structure of the circle model changes when we append the logit to it. It may help here to think of restaurants, so that one source of differentiation is geographical, and this is a source of localization of competition. Ceteris paribus, localized differentiation is more important the greater the transport rate, t. On the other hand, each restaurant provides a different menu, different ambience, decoration, and style. For this second source of differentiation, the *i.i.d.* assumption means that knowing a consumer's preferred restaurant style tells us nothing about her next preferred restaurant style, etc. Furthermore, under the logit specification, the relative percentages of consumers visiting the fish and chip shop over the pizza parlor are independent of the price at the burger joint. The intensity of this taste component is measured by the parameter μ, which is proportional to the standard deviation of the taste density function. Global differentiation is more important, ceteris paribus, the greater is μ.

We treat geographic and product matches as independent factors in tastes. As in econometric models, we sum the two match types. Hence we write the total firm match as:

$$\epsilon_{ij} = -t|z_j - z_i| + \mu e_{ij}, i = 1...n. \tag{7}$$

Each consumer type is described by a position z_j on the circle as well as a vector of product specific match values e_{ij} that are double exponentially distributed.[5] The model is consistent with either intrinsic consumer preferences (each consumer draws her product match values once and for all, and always buys from the same firm), or else fluctuating preferences (a new set of product match values is drawn each shopping trip, so the consumer tends to buy from different firms).

The probability that a consumer located at z_j buys from firm i is $\mathbb{P}_i(z_j) = \text{Prob}(V_{ij} \geq V_{kj}$, $k = 1...n)$. Using (1) and (7) with the double exponential distribution leads to the logit specification

[5]The corresponding density of types is $(N/L)\prod_{i=1}^{n} f(e_i)$, with $f(e_i) = \exp[-e_i]\exp[-\exp(e_i)]$.

for any z so:

$$\mathbb{P}_i(z) = \frac{\exp[(v(p_i) - t|z - z_i|)/\mu]}{\sum_{k=1}^{n} \exp\left[\left(v\left(p_k\right) - t|z - z_k|\right)/\mu\right]}, i = 1...n. \tag{8}$$

In this setting, μ represents the weight given to product matches, and t the weight on distance. For $t > 0$, and $\mu \to 0$, each firm is in competition with only its two neighbors in geographic space (pure local differentiation). As μ rises from zero, firms start to attract consumers from beyond their neighbors, and so come into contact with firms farther away. However, direct competition with other firms is lower the farther away they are (farther down the chain). Spatial differentiation is less important the higher is μ/t, with more overlapping along the chain of firms. When μ/t is large enough, the spatial chain aspect essentially disappears, and all products become equally good substitutes (pure global differentiation). If both μ and t go to zero, then all products become perfectly homogenous and price goes to marginal cost (the standard Bertrand result). Note though that μ and t enter the choice probability function (8) in different ways: in particular, higher μ decreases price sensitivity as well as distance sensitivity whereas lower t only reduces distance sensitivity. These differential effects lead to qualitatively different comparative static effects for these two variables, as we shall see in the sections below.

We measure the *effective product variety* available to consumers by $\phi = \exp(-\frac{tL}{n\mu})$. This index is zero if products are completely undifferentiated ($\mu = 0$); for $t > 0$, $\phi = 0$ corresponds to pure localized competition. The index rises with n (more choice) and with μ (more intrinsic preference for variety), and falls with tL (more spatial friction). When products are very differentiated or when the transport rate is very small, each firm competes symmetrically with all others and competition is purely global. In this case $\phi = 1$. Hence ϕ provides a measure of the degree of global competition in the market.

The formulation (8) was first proposed by de Palma et al. (1985) and was there applied to firm location in a linear segment; later developments are described in Anderson et al. (1992, Ch 9). This earlier work treats $\nu(p_i) = -p_i$, so that each consumer buys one unit of the good. Much more importantly, the earlier work is either an explicit duopoly analysis, or else essentially duopoly in that agglomeration equilibria are derived by showing that one firm does not wish to deviate from the midpoint location occupied by the remaining $(n-1)$ firms. The present paper analyzes a full-fledged oligopoly model and, furthermore, allows for an endogenous number of firms.

4. MARKET EQUILIBRIUM

We can now derive the equilibrium price for the oligopoly model. Assume there is an even number of firms and let the number of pairs of firms be $m \geq 1$.[6] From the expression for the symmetric price equilibrium (5), we must derive the market share derivative S', evaluated at a symmetric solution.

The market share expression is given by $S_i = \frac{2}{L}\int_0^{L/2} \mathbb{P}_i(z)dz$, where $\mathbb{P}_i(z)$ is given by (8).[7] Hence, at the symmetric candidate equilibrium,

$$S' = \frac{2}{L\mu}\int_0^{L/2} \hat{\mathbb{P}}_i(z)(1 - \hat{\mathbb{P}}_i(z))dz \tag{9}$$

where (using (8)),

$$\hat{\mathbb{P}}_i(z) \equiv \exp[-t|z - z_i|/\mu] / \sum_{k=1}^{n} \exp[-t|z - z_k|/\mu]. \tag{10}$$

To evaluate S' we must first rewrite $\hat{\mathbb{P}}_i(z)$: without loss of generality, we may consider firm 0,

[6]The analysis that follows can be reformulated for an odd number of firms, but there are no qualitative changes.

[7]For $t > 0$, market share S_i and firm demand $x(p_i)S_i$ are no longer constrained by the IIA property discussed in the previous section (in contrast to the standard logit and CES models).

located at 12 o'clock. Let - $\equiv L/2m$ (the interfirm distance - there are $2m$ firms) and recall that $\phi \equiv \exp(-t\text{-}\ /\mu) \in (0,1)$. Let $z \in (k\text{-}\ ,(k+1)\text{-}\ )$, for some $k = 0...m-1$. Any consumer at z patronizing a firm $l = k+1...k+m$ will "travel" clockwise to get there, so $|z - z_\ell| = (\ell\text{-}\ \ - z)$ for such firms. Travel to the remaining firms is counter-clockwise, involving distances $z - \ell\text{-}$ for $\ell = 1...k$ and $z + (2m-\ell)\text{-}$ for firms $l = k+m+1...2m$. After summation,

$$\hat{\mathbb{P}}_0(z) = \{\zeta[\phi^k \exp[\frac{2tz}{\mu}] + \phi^{-(k+1)}]\}^{-1}, \tag{11}$$

where $\zeta \equiv \phi(1-\phi^m)/(1-\phi)$. Given this expression,

$$\int_k^{(k+1)} \hat{\mathbb{P}}_0^2(z)dz = \frac{\mu\phi^{2(k+1)}}{2t\zeta^2}\left[\frac{(\phi-1)}{(\phi+1)} + \frac{t\text{-}}{\mu}\right], \tag{12}$$

and

$$\int_0^{L/2} \hat{\mathbb{P}}_0^2(z)dz = \frac{\mu}{2t\zeta^2}\left[\frac{\phi-1}{\phi+1} + \frac{t\text{-}}{\mu}\right]\frac{(1-\phi^{2m})\phi^2}{(1-\phi^2)}. \tag{13}$$

We also know $\int_0^{L/2} \hat{\mathbb{P}}_0(z)dz = \frac{\ }{2}$ (by symmetry, firms share the market equally), so the required expression is $S' = 1/(n\psi(n))$, where

$$\psi(n) = \frac{\mu(1-\phi^m)(1+\phi)^2 \ln\ \phi}{2(\phi-\phi^m)(1+\phi)\ln\ \phi - (1+\phi^m)(1-\phi)^2}, \tag{14}$$

and we recall $n = 2m$. Therefore, the candidate symmetric equilibrium price given by (5) has the explicit form

$$p^* = [(1-\alpha)\psi(n)]^{1/(1-\alpha)}, \tag{15}$$

which is strictly positive since $\phi \in (0,1)$. From (6), the corresponding profit per firm is:

$$\Pi^*(n) = \frac{N(1-\alpha)\psi(n)}{n} - K. \tag{16}$$

Proving that (15) is indeed an equilibrium is not a simple matter. Standard existence techniques require showing either that profits are quasiconcave or that the game is supermodular. The oligopoly game here is not supermodular, nor are profits always quasiconcave.[8] There are important cases when they are quasiconcave, however, and then existence is ensured.

First, for duopoly ($m = 1$), the results of Caplin and Nalebuff (1991) can be applied: the profit function is always quasiconcave and existence is ensured. Second, if μ is small enough, the fact that the profit function and candidate equilibrium price are continuous in μ implies that profit remains quasiconcave because it is strictly concave when $\mu = 0$ (see Anderson and de Palma, 1995, p.38, for a more formal argument and Appendix 1 for a proof that profit is concave - and so equilibrium exists - for the circle model with price-sensitive demand). Hence p^* as given by (15) is an equilibrium for μ small enough.

Third, for μ large enough, we can also show analytically that profit is quasiconcave. The argument has some intrinsic interest and is given in Appendix 2, where we show that profit is quasiconcave (and equilibrium exists) for $\phi = \exp(-t\text{-}\ /\mu) \geq 1/3$ (or, equivalently, $tL/m\mu \leq 1.1$).

[8] Milgrom and Roberts (1990) prove existence by showing that the logarithm of profit for the logit model has the desired property, that marginal profit increases with rivals' prices, so reaction functions can only jump up. This method will not work here. (Since demand is the integral over space of logit demands, taking logarithms or other transformations does not simplify matters.) Indeed, for small μ the reaction function exhibits downward jumps, and so the game cannot be supermodular. To see this, consider the circle model ($\mu = 0$) with price-sensitive demands. If the common price of the rivals is high enough, the best reply is to price undercut. This example also shows that the profit functions are not generally quasiconcave for arbitrary levels of the rivals' prices. However, at the candidate equilibrium price (t/Ln for the circle model where $\mu = 0$) profit is actually concave where positive because undercutting entails a zero price (see Appendix 1). This means that it is not possible to get a general quasiconcavity result simply because profit is *not* quasiconcave for all levels of rivals' price.

The structure of this proof is to first consider the profit of firm 0 at a point z as a function of p_0. This point-profit is concave up to a critical price $p_0^I(z)$ which exceeds the price that maximizes the point-profit, and profit is falling beyond $p_0^I(z)$. When differentiation is relatively global, the point-profit functions are relatively "close" to each other, so that the one that has the greatest maximizing price (*i.e.*, for $z = 0$) is decreasing for prices such that the one for which $p_0^I(z)$ is smallest (*i.e.*, for $z = L/2$) is still concave (see Figure 3 in Appendix 2). Hence the total profit function is concave and then decreasing, and so is quasiconcave. The proof finds the critical value (the critical "closeness") as $\phi = 1/3$ or $\mu \approx 0.9tL/m$.[9]

We have run several numerical computations of the behavior of the profit function for values of μ that the analytic arguments do not cover. In all cases, the profit functions are embarrassingly quasiconcave (although very complicated when explicitly integrated). A typical example is given in Figure 1.

Figure 1. Behavior of profit function, $n = 4$, $t = 1$, $\alpha = 0$, and $\mu = 0.08293$.

To sum up, the existence issue revolves around showing that the candidate price as given by (15) cannot be improved upon by unilateral deviations. For duopoly, profit functions are quasiconcave for all possible prices so (15) then constitutes an equilibrium. For more than two firms, (15) describes an equilibrium if μ is either small enough or else above a critical value that nevertheless still corresponds to a fairly localized degree of competition (and equilibrium exists for all higher degrees of global competition). The proof of the latter property constitutes a contribution to

[9] To get an idea of the extent of localization consistent with the bound $\phi = 1/3$ given from the existence proof, recall that under pure local differentiation ($\mu = 0$), all consumers in the interval $[0, -\ /2]$ buy only from firm 0. For $\phi = 1/3$ the fraction of consumers in this interval buying from firm 0 is $\frac{2}{3}\left[1 + \ln 3\ \ \ln \frac{2}{3}\right] / \left(1 - \left(\frac{1}{3}\right)^m\right)$, which approaches .37 as m gets large. One way to think of this number is that the consumers closest to a restaurant buy from it over one third of the time. In that sense competition seems quite localized. For higher values of ϕ, existence is guaranteed, and the spatial distribution of consumers is more even.

the literature on equilibrium existence in spatial markets since it exploits the spatial structure by looking at profit functions at each point in space and then aggregating to prove overall profit quasiconcavity. For intermediate values of μ, for which we have no analytic proof, all simulations indicate that profit functions are quasiconcave (given all other firms set the price (15)), which is a stronger property than is needed for equilibrium existence. Thus we have no counter-examples to equilibrium existence, and we conjecture that (15) is indeed an equilibrium for all parameter values.

The limit cases of the equilibrium price (15) correspond to well-known models. As $\mu \to 0$, (14) reduces to tL/n, the price equilibrium for the standard circle model (when $\alpha = 0$). For $t \to 0$, it can be shown that (14) converges towards $\frac{\mu n}{n-1}$, which is consistent with the standard aspatial logit result (see Anderson, de Palma, and Thisse 1992, for the case of $\alpha = 0$).[10] Finally, if $n \to \infty$, with L finite, which can be construed as the case of monopolistic competition, from (14) we have $\psi = \mu$. (The same limit arises when $n \to \infty$ for the aspatial logit - transport costs become irrelevant when more and more firms are crowded into a fixed product space.)[11]

5. FREE ENTRY EQUILIBRIUM

The zero-profit equilibrium is that envisaged by Chamberlin, and we shall refer to this as the "long-run" equilibrium. The associated number of firms is characterized (from (16)) as the solution, n,

[10]Another interesting limit expression corresponds to an infinite line model which has appeared in the spatial literature (see Eaton and Wooders, 1985) as an alternative method of circumventing boundary problems. The idea is again to treat equally spaced firms: this is captured by holding L/n constant while letting both L and n tend to infinity. Thus the circumference of the circle becomes arbitrarily large, and, for $n \to \infty$, expression (14) becomes $\psi(n) = \frac{\mu(1+\phi)^2 \ln \ \phi}{2\phi(1+\phi)\ln \ \phi - (1-\phi)^2}$,which again reduces to tl/μ as μ goes to zero.

[11]The result that price exceed marginal cost can be ascribed to the behavior of the upper tail of the double exponential distribution (see Perloff and Salop, 1985). For other distributions with "thinner" tails (such as the normal, which leads to the probit model), price converges to marginal cost when n is very large. In our case, a mark-up is sustained because brands retail loyal consumers whose valuations of their most preferred products are sufficiently higher than their next most preferred product even in the limit.

to

$$\frac{N}{n}[(1-\alpha)\psi(n)] = K. \tag{17}$$

The zero-profit characterization can be criticized on two counts. First, because the number of firms should reasonably be an integer, it is an approximation, although a relatively innocuous one when the number of firms is large. More importantly, Eaton and Lipsey (1978) argued that pure profits can be sustained in free-entry equilibrium because an entrant must fit into a gap between existing firms, and thus earn substantially lower revenue than the incumbents. Here we implicitly assume (as do Salop, 1979, and many other authors) that firms relocate to a symmetric position subsequent to entry.

To get a better intuition for the comparative statics that follow, it is worth spending some time discussing how the short-run equilibrium changes with n. As one might expect, the equilibrium price is decreasing with n: more firms means more competition for consumers. This result is readily shown by differentiating (14) (see Anderson and de Palma, 1995). Revenue per firm falls as n rises because prices are lower and aggregate demand is inelastic, so a smaller total revenue pie is shared among more firms. Hence the long run equilibrium is uniquely determined.

The effects of an increase in n on output per firm (which we henceforth refer to as firm size) are more involved. Clearly total output, $Np^{-\alpha}$, goes up because price falls, but this is shared among more firms. It turns out that *firm size rises if aggregate demand is sufficiently elastic* (α is large enough). To illustrate, consider first $\mu = 0$ (circle), where equilibrium size is $\frac{N}{n}[\frac{(1-\alpha)tL}{n}]^{-\alpha(1-\alpha)}$, which is proportional to $n^{(2\alpha-1)/(1-\alpha)}$. Clearly firm size increases with n if $\alpha > 1/2$. If $t = 0$ (with $\mu > 0$), size is proportional to $[(n-1)^{\alpha}/n]^{1/(1-\alpha)}$ and increases with n if $\alpha > 1 - 1/n$, which exceeds $1/2$ as long as there are more than two firms. In this sense, firm size is more likely to rise when competition is more localized. When competition is global, we just showed that there is a critical value of n (equal to $1/(1-\alpha)$) such that size increases with the number of firms up to this value, and decreases beyond it. We should not conclude from this that firm size will rise and then fall (if the demand structure is sufficiently global) in an industry where the number of firms grows over time. It is also necessary to explain why the number of firms has changed. For example, an increase in transport costs, taste for variety, or population, or a decrease in costs could all lead to more firms in the long run, but changes in these variables also have direct (short run) effects on firm size. These issues are addressed in the next four sections.

We have also compared equilibrium and optimum numbers of firms. There is always overentry. This result is not too surprising (and we do not dwell on it here), since the circle model involves massive overentry: for $\alpha = 0$, the equilibrium number is twice the optimum one, and for $t = 0$, the numbers are about equal (but just on the overentry side). As t increases from zero, the equilibrium and optimum numbers diverge more. Loosely, larger transport costs lead to less effective competition (competition is more localized), and the high prices attract too many firms.[12]

6. TRANSPORT IMPROVEMENTS

Transport costs have decreased dramatically this century. In the city, there was the development of mass transit. After 1945 came massive investment in road infrastructure coupled with a spectacular rise in car ownership. Even earlier, the costs of shipping goods was greatly reduced with the use of canals and then railways. Technological developments like bottling, canning, and pasteurization

[12]See Deneckere and Rothschild (1992) for more on the comparison between pure local and pure global competition. Our model provides a path from one extreme to the other. Anderson, de Palma, and Nesterov (1995) showed that there is overentry for $t = 0$ and $\alpha < 1$, for all logconcave match distributions (logconcavity being a sufficient condition for a price equilibrium to exist, as per Caplin and Nalebuff, 1991).

meant it was possible to ship goods over long distances without spoilage. The consequences for industry structure have been profound.

In our model, a fall in t induces some consumers to travel farther to find a better product match (this does not happen in the standard circle model). It also means that a rival's customer is easier to sway with a price cut. Both effects create more effective competition, and this reduces prices.[13] With lower prices, profits fall but firm size rises. Hence lower transport costs lead to larger firms in the short run and exit in the long run. As firms exit the industry, competition is weakened and prices rise somewhat. They do not rise so much that the exit effect offsets the initial effect. If they did there would be fewer firms selling at higher prices, which is not in accord with the zero-profit equilibrium. *The long-run equilibrium therefore has fewer and larger firms, with lower prices.* Supermarkets tend to replace Mom-and-Pop grocery stores, small local brewers give way to large multinationals.

The index of effective variety, $\phi = \exp(-tL/\mu n)$, clearly rises in the short run because consumers can travel more cheaply to find products they prefer. In the long run, the number of firms falls, so there is a priori some ambiguity as to the overall effect. A sufficient condition for ϕ to rise is that ψ does not fall as much as t in percentage terms (it may help for the intuition to recall that ψ is simply the equilibrium price when $\alpha = 0$). To see this, suppose t is halved and ψ falls by half or less. In the short run, revenues fall by half or less (see (16)). If prices were held fixed and half the firms exited, revenues would return to at least their original level. But prices are not fixed when firms exit, they rise. Thus it must be that fewer than half the firms exit. Accordingly, t/n falls and ϕ rises. This argument does not always apply because certain parameter values admit a

[13] A formal derivation of the result that $\partial\psi/\partial t > 0$ is given in Anderson and de Palma, 1995.

percentage fall in ψ greater than the percentage fall in t. This only happens for μ small.[14] Suffice it to conclude that the usual case (if not the only case, we have no counter-example) is that, as expected, effective variety rises in the long run when transport costs fall.

7. WHEN CONSUMERS BECOME MORE CHOOSY

Subsequent to the large decrease in transport costs, casual observation suggests that products have become more differentiated over time and consumers are increasingly willing to pay for products which better fit their preferences. What suits one consumer may be anathema to another (*e.g.* sushi, tofu, frogs' legs, or horsesteak): products are horizontally differentiated. The number of types of computer software seems to be growing without bound, ethnic restaurants flourish, and more and more types of beer are available in bars. Recent advances in Marketing emphasize the importance of tailoring goods to individual tastes and specific market niches, and technological advances have made it easy to produce goods which are more differentiated. It does not matter whether products are actually more differentiated, whether consumers just perceive them to be more differentiated, or if consumers are just more choosy. All these changes are described in our model by an increase in the parameter μ. Note that increases in μ are not equivalent to decreases in t (see (14) and (15) and note the factor μ in the numerator of (14) that is not offset by a t). While higher μ does reduce distance sensitivity, it also reduces price sensitivity, which has important effects on the comparative static result.

The first (and perhaps most interesting) result is that *equilibrium price does not necessarily rise with* μ. A standard result in the literature on product differentiation is that greater product

[14] Indeed, simulations indicate that this can happen only around the "dip" in the equilibrium price as a function of μ, described in the next section.

heterogeneity leads to higher equilibrium prices (see for example Anderson, de Palma, and Thisse, 1992, ch. 6) because firms have more market power if consumers have more intense preferences for particular products. This is not necessarily true in our spatially extended model. It can be shown for $n > 2$ that $\delta p^*/\delta\mu < 0$ for μ small enough and $\delta p^*/\delta\mu > 0$ for μ large enough.[15] However, $\delta p^*/\delta\mu < 0$ for $n = 2$. A typical pattern of the equilibrium price as a function of μ is represented in Figure 2.

Figure 2. Equilibrium price, $n = 4,\ t = 1,\ \alpha = 0.$

The intriguing result is that competition can become tougher when products become more differentiated. The intuition for this result should (and does) rely on the spatial structure of the model. If μ is small, firms behave as in the pure spatial model and each firm competes mainly with its immediate neighbors. All the action takes place at the frontier between the (almost perfectly delineated) market areas. Firms compete primarily for the "undecided" consumers located halfway between each pair of adjacent firms. As μ rises, consumers start to entertain new possibilities (new cafes for the right beer or cocktail, or the perfect environment). Competition is more intense as a result: a cafe can more easily attract customers from the other side of town by slightly cutting its price because some customers there really like this cafe. When μ rises more, the standard effect dominates and price competition is relaxed. This intuition does not apply if $n = 2$ because each firm competes directly with all its rivals (only one firm here) even when $\mu = 0$.

In summary, as μ rises from zero, market boundaries become blurred and each firm finds rivals encroaching in its previously protected spatial market. As the barriers of spatial separation crumble, prices fall both to retain customers and to attract customers from farther afield. At the same time,

[15] Figure 1 is drawn for $\mu = 0.08293$, which corresponds to the bottom of the price dip for $n = 4$ and $t = 1$.

increasing μ creates greater brand loyalty in the nonspatial dimension, which tends to increase equilibrium prices. For large enough μ the latter effect comes to dominate. It is instructive to compare these results for those for a decrease in t (see the preceding section). For a decrease in t, customers tend to travel further and it is easier to sway a customer from a rival firm, leading to more effective competition and lower prices. An increase in μ also induces customers to travel further (and so they are more susceptible to capture), but (ceteris paribus) it becomes less easy to sway a customer from a rival since they become less price sensitive. The impact of the second effect dominates when μ is large and consumers choose their idiosyncratic product with scant regard to transport costs.

In the short run, firm size rises with μ when it is small (if and only if $n > 2$)) and decreases if it is large enough. The index of effective variety ϕ, unambiguously rises in the short run: competition becomes more global. Now consider the long run. In the first case ($\delta p^* / \delta \mu < 0$), profit decreases with μ, and some firms will leave. This situation is similar to a reduction in the transport cost rate discussed in the previous section, and results in fewer and larger firms selling at lower prices. The change in the index ϕ is more intricate in this case. Products are more differentiated, but there are fewer firms. This leaves open the possibility that effective variety may fall with greater taste for variety in the long-run.

If μ is large enough that $\delta p^* / \delta \mu > 0$, firm profits increase and the number of firms increases as well. Entry will contribute to decrease the price level somewhat (but not as low as before the change in μ).[16] Therefore a rise in μ will in this case increase the price level and decrease firm size in the long run. The index ϕ will increase for two reasons: greater appreciation of variety and more

[16]Since there are more firms, prices must be higher to support them.

varieties provided. Competition is much more global as a result.[17]

One should not infer that the taste for variety has increased from the observation that there are more firms. Indeed, even if we are not in the perverse part of the parameter range, there are other reasons for increases in the number of firms. An obvious one treated in the next section is that population has risen.

8. POPULATION GROWTH

One cannot ignore the explosive population growth in the world and especially in its cities. One would expect that this would lead to more and larger firms. This is indeed true in our model. The mechanism is as follows. In the short run, price is independent of population, N, since replicating consumers simply adds demand curves horizontally. Firm size rises proportionally to the consumer population, as do gross profits. In the long run, more firms enter, driving down prices, but *the increase in the number of firms is less than proportional to the population increase.* (The population increase would support a proportional increase in the number of firms if prices did not fall with n. The increase in competition explains the result.) Output per firm rises because price falls, using the equality of fixed cost to firm revenue at the zero-profit equilibrium. Our index of effective variety, ϕ, rises as population grows because there is more choice. There are more things to do in denser cities because the larger population supports more "blocks" of fixed cost. In a similar vein, this is one of the main sources of gains from trade when products are differentiated.

In the analysis above, population was increased without any concurrent rise in the geographical

[17] Another comparative static result of interest corresponds to increasing both μ and t in equal proportions. This increases total product differentiation, while leaving the "mix" between global and local differentiation unchanged. In the short run, ϕ is unaffected by such a change, so the only thing that happens (see (14)) is that price rises proportionately. In the long run firms enter and are smaller, and effective variety rises.

size of the city. To account for area differences as well as population differences, let L rise the same percentage amount as N so the density does not change as the city size increases. The short run gives higher prices (since this is equivalent to a rise in t and a rise in N). More firms enter, but in a lower proportion to the rise in L and N. To see this, suppose that the city doubled (but kept the same density). Then 2n firms would make negative profits because firms face more competition which drives prices down.[18] Thus prices are lower, and firms are larger both because of the price effect and because the number of consumers has risen more than the number of firms. The impact on ϕ is rather interesting. Since L/n rises, ϕ falls, suggesting that *larger cities actually may have less effective variety*. This is so because competition is more intense, ceteris paribus, *in large cities, so the market supports fewer (but larger) stores per consumer*. An important caveat is that larger cities are more likely to have larger population densities, and, as we have seen for the reasons on N alone, this effect per se increases effective variety. In the Salop model, increasing city size as above merely replicates the initial structure. A city that is twice as large (with the same consumer density) has twice the number of firms. There is no diversity effect.

9. COST CHANGES

It is more difficult to take a stand on how costs have changed over the years. If cottage industries give way to mass production, the degree of increasing returns to scale rises. We capture this by supposing fixed costs, K, rise, and average variable costs fall. Let us first treat the rise in fixed costs in isolation (for example, store rents rise in city). Clearly nothing happens in the short run, except that firms make losses. Some exit, and prices rise in the long run. The effects on firm size

[18]When $\mu = 0$ (Salop's circle case) there is no such price effect, so increasing size simply increases firm numbers proportionately.

are ambiguous, as noted in Section 5. The more surprising case is that *firms can actually shrink as others exit the industry.* This can happen if aggregate demand is sufficiently elastic, and is more likely when demand is local. Exiting firms cause price to rise, and the price rise is greater when competition is mainly local because firms have more local market power when rivals leave. The price rise can be so large that it offsets the larger market share effect of exit. High rent areas may have small shops not only because shops economize on rent payments by having a small surface area, but also because the low level of competition leads to high prices and low demand.

Marginal production costs can be introduced into the model in several ways. The easiest one is to simply reinterpret the parameter α as the elasticity of conditional demand with respect to the markup. That is, let $\alpha = -x'(p)[p-c]/x(p)$, or indeed, $x(p) = (p-c)^{-\alpha}$. The analysis so far is unchanged, except prices are interpreted as equilibrium mark-ups. This approach is rather ad-hoc. It also does not yield the CES as a special case, and one of our objectives is to integrate the major models of product differentiation. A slightly different assumption does yield the CES as a limit case. Suppose $\alpha \equiv -x'(p)p/x(p)$ as before, with $v(p) = \frac{1-p^{1-\alpha}}{1-\alpha}$ and $\alpha \in [0,1)$, and let $c > 0$ be marginal production cost. The CES model is given as the limit case $\alpha \to 1$ (so $v(p) = -\ln\ p$) and $t \to 0$. In this case the demand for product i is

$$Nx(p_i)S_i = \frac{N}{p_i}\frac{\exp[(-\ln\ p_i)/\mu]}{\sum_{k=1}^{n}\exp\left[(-\ln p_k)/\mu\right]} = \frac{Np_i^{-1/\mu-1}}{\sum_{k=1}^{n}p_k^{-1/\mu}},\ i = 1...n, \tag{18}$$

where S_i is given by (8).[19]

[19]This demand system can also be derived from a representative consumer endowed with the aggregate income and CES preferences

$$U = N\ln(\sum_{k=1}^{n} x_k^{\rho})^{1/\rho} + x_0, \tag{9.1}$$

where $\rho \equiv \frac{1}{1+\mu}$ and x_0 is consumption of the numeraire. See Anderson et al. (1992) for a similar derivation of an

Returning to the general case corresponding to Section 4 with positive marginal cost, firm i's profit is now written as $\Pi_i = N(p_i - c)x(p_i)S_i - K$, so the candidate symmetric equilibrium price (given n) is now the implicit solution to:

$$G(p) \equiv -p^{1-\alpha}(p-c)S'n + p(1-\alpha) + \alpha c = 0, \tag{19}$$

where the expression for S' is the same as in Section 2. The solution, $p^* > c$, is unique since G is continuous in p and $G' < 0$ when $G(p) = 0$.[20] Anything that raises $S'n$ decreases p^*. In particular, the earlier comparative static results hold (recall that $S'n = 1/\psi(n)$). The only result that does not hold is the ambiguity of firm size with respect to entry (see Section 5). In the present version, fewer firms (due to an increase in K say) lead to larger firm size for both localized and global competition. This suggests the model is sensitive to the way in which marginal costs are added.

The effects of lower marginal cost are straightforward. From (19), price falls (so firm size increases) and the mark-up rises in the short run. Firms enter, and the long-run equilibrium therefore involves lower prices and larger firms (and hence lower mark-ups). Taking these results in conjunction with the results for increased fixed costs, the adoption of technologies with increased returns to scale leads to larger firms. The effects on price and firm numbers are ambiguous, but a sufficient condition for price to fall is that the number of firms does not fall.

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alternative CES form.

[20] Differentiating (19) yields $G' = (1-\alpha) - S'n((2-\alpha)p^{1-\alpha} - (1-\alpha)cp^{-\alpha})$. Evaluating at $G(p) = 0$ gives $G'(p^*) = (1-\alpha) - \frac{[p(1-\alpha)+\alpha c][(2-\alpha)p-(1-\alpha)c]}{(p-c)p}$ which has the sign of $\alpha(1-\alpha)k^2 - \alpha(3-2\alpha)k - (1-\alpha)^2$, where $k \equiv c/p \in (0,1)$. This latter expression is convex, and negative at $k=0$. At $k=1$ it is also negative and is therefore negative throughout.

10. CONCLUSIONS

The approach presented in this paper provides a synthesis of major existing models of product differentiation. Special cases yield the CES, the logit, the circle model, and a circle model with price sensitive demand. The integrative framework enables us to link models that are rooted in very different primitives on consumer behavior (*e.g.* representative consumer and spatial models).

The benefit of the integrative framework is that it allows us to address issues that cannot be properly formulated otherwise. For instance, the standard circle model does not admit non-spatial product differentiation, so it cannot be used to tell us what happens when consumer taste for variety rises. Likewise, the CES model is not appropriate for analyzing reductions in transport costs. Realistically, both spatial location and product differentiation interact in determining market structure. The interaction between the two is fundamental, and not simply the sum of the two components. For example, higher taste for variety raises price when there is no geographical differentiation ($t = 0$), but may decrease it when t is positive.

Although the model is strictly one of shopping behavior (but note that it applies equally well to industrial location for $\alpha = 0$), it is tempting to interpret the qualitative results in terms of the evolution of industrial structure. The main changes over the past two hundred years are a huge reduction in transport costs, an increase in consumer taste for diversity, population growth, and adoption of production techniques with greater returns-to-scale. That is, a drop in t, increase in μ, increase in N, rise in K, and fall in c. Over the same period, industrial structure has changed from many small-scale producers (a brewer in every town) and low effective product variety to few large-scale producers with low real prices, and a large effective variety. That is, a decrease in n, increase in $N\mathbb{P}x$, fall in p^*, and increase in ϕ.

These changes are clearly related, although several of what we treat as exogenous changes work in opposite directions to the changes in the endogenous variables. What this means in effect is that some effects outweigh the others. For example, if the number of firms is observed to fall, this should be because the population growth effect is outweighed by the increasing returns effect (higher K), and lower transport costs outweigh any effect from increased preference for variety. It is worth noting recent developments in the brewing industry: the advent of micro-breweries in North America, and the revitalization of small breweries in the UK (following the Campaign for Real Ale in the 1970's). This is what is expected from more taste for product variety (higher μ).

	Fixed number of Firms					Long Run		
	Firm size	Price	Profit	ϕ	n	Firm size	Price	ϕ
$n\uparrow$	?	$-$	$-$	$+$	*Not applicable*			
$t\downarrow$	$+$	$-$	$-$	$+$	$-$	$+$	$-$	$+$
$\mu\uparrow$ μ low	$+$	$-$	$-$	$+$	$-$	$+$	$-$	?
$\mu\uparrow$ μ high	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$+$
$N\uparrow$	$+$	0	$+$	$+$	$+$	$+$	$+$	$+$
$\%\ N\uparrow = \%\ L\uparrow$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$-$
$c\downarrow$	$+$	$-$	$+$	0	$+$	$+$	$-$	$+$
$K\uparrow$	0	0	$-$	0	$-$	?	$+$	$-$

The comparative static properties of the model are summarized in the table above. The first entry is an increase in the number of firms ($n\uparrow$), and is useful in understanding the long-run results that follow. The next effects considered loosely follow the major changes in fundamentals. These are lower transport cost ($t\downarrow$), larger taste for variety ($\mu\uparrow$), larger population size ($N\uparrow$), and an increase in the geographical market size proportional to the population increase ($\%N\uparrow = \%\ L\uparrow$).

Finally, the cost changes we consider are a lower marginal cost ($c \downarrow$) and a higher fixed cost ($K \uparrow$).

It would be interesting to calibrate the model to simulate either a study of an industry over time, or else to undertake a comparison across different industries. Empirical work would be most welcome in evaluating the size of the various effects. In its present symmetric state, the model is unlikely to be directly appropriate for specific industry studies. There are several straightforward ways to introduce asymmetries into the model. First one could use a linear market segment instead of the circle if analyzing a pure characteristics model; for a geographic model of firm location a two-dimensional space is appropriate. The symmetry of locations could also be relaxed. Further asymmetries can be introduced using quality variables for products. Finally, where multiproduct firms are important, let us note that Anderson, de Palma, and Thisse (1992) have formulated a multiproduct firm model based on the nested logit that could prove useful in applications.

One criticism of the model is the zero-profit assumption used to characterize long-run equilibrium. The insight of Eaton and Lipsey (1978) was that pure profits may be earned when space matters because a new entrant must fit into a niche between existing firms. Their argument suggests that the potential for pure profit should be greatest when the structure of differentiation is completely local. We tested this conjecture in Anderson and de Palma (1995). We showed that the amount of profit that can be shielded from entry is first *increasing* with product taste diversity (μ) and then decreasing. The initial counterintuitive result is due to the behavior of the equilibrium price for low values of μ (see Section 7): it becomes easier to keep entrants out as μ rises at first because price falls. One would like to check out these types of questions more generally.

Finally, the model provides a sparsely parameterized version of a Chamberlinian setting, but with oligopoly rather than monopolistic competition. (That is, n is not necessarily infinite.) As the intuitive description of the comparative static properties in the paper makes clear, the properties are more general than our particular parameterization. We hope that this description will be useful both in Industrial Organization and in Economic Geography.

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APPENDIX 1. Proof of equilibrium existence and uniqueness for the circle model with price-sensitive individual demands.

We first consider prices such that no firm is undercut. Using (1), the consumer $\hat{z} \in (z_i, z_{i+1})$ indifferent between i and $i+1$ is at

$$\hat{z} = \frac{1}{2t}\frac{(p_{i+1}^{1-\alpha} - p_i^{1-\alpha})}{1-\alpha} + \frac{z_{i+1}+z_i}{2}$$

so $d\hat{z}/dp_i = -p_i^{-\alpha}/2t$. Similarly, let $\bar{z} \in (z_{i-1}, z_i)$ be the consumer indifferent between i and $i-1$, with $d\bar{z}/dp_i = -d\hat{z}/dp_i$. Firm i's profit is $\Pi_i = Np_i^{1-\alpha}(\hat{z}-\bar{z})$, so $\frac{1}{N}\frac{\partial \Pi_i}{\partial p_i} = (1-\alpha)p_i^{-\alpha}(\hat{z}-\bar{z}) - p_i^{1-2\alpha}/t$. Hence

$$\frac{p_i^{\alpha}}{N}\frac{\partial \Pi_i}{\partial p_i} = \frac{1}{2t}\left(p_{i+1}^{1-\alpha} - 2p_i^{1-\alpha} + p_{i-1}^{1-\alpha}\right) + \frac{L}{n}(1-\alpha) - p_i^{1-\alpha}/t \qquad \text{(A1.1)}$$

All firms must be active at any equilibrium, so that (A1.1) is necessarily zero for all $i = 1...n$, *i.e.*

$$p_{i+1}^{1-\alpha} + p_{i-1}^{1-\alpha} - 2p_i^{1-\alpha} = -\frac{2tL}{n}(1-\alpha) + 2p_i^{1-\alpha}, \quad i = 1...n. \qquad \text{(A1.2)}$$

Summing over $i = 1...n$, yields $tL(1-\alpha) = \sum_{i=1}^{n} p_i^{1-\alpha}$, so (A1.2) becomes

$$p_{i+1}^{1-\alpha} + p_{i-1}^{1-\alpha} - 2p_i^{1-\alpha} = 2\left(p_i^{1-\alpha} - \frac{1}{n}\sum_{k=1}^{n} p_k^{1-\alpha}\right). \qquad \text{(A1.3)}$$

Select the lowest price firm. Then the LHS of (A1.3) is non-negative, but the RHS is non-positive. The only equilibrium candidate is thus the symmetric one.

To prove this is an equilibrium we first show profit is quasiconcave for prices such that i does not undercut, and second that undercutting neighbors is never profitable. From (A1.1),

$$\frac{\partial}{\partial p_i}\left(\frac{p_i^{\alpha}}{N}\frac{\partial \Pi_i}{\partial p_i}\right) = \left(\alpha p_i^{\alpha-1}\frac{\partial \Pi_i}{\partial p_i} + p_i^{\alpha}\frac{\partial^2 \Pi_i}{\partial p^2{}_i}\right) < 0,$$

so the profit function is necessarily strictly concave whenever $\partial\Pi_i/\partial p_i = 0$ and is hence strictly quasiconcave.

To undercut its neighbors, firm i must set a price p_i^u satisfying $\frac{(p_i^u)^{1-\alpha}}{1-\alpha} = \frac{(p^*)^{1-\alpha}}{1-\alpha} - \frac{tL}{n}$. Substituting $p^* = [\frac{(1-\alpha)tL}{n}]^{1/(1-\alpha)}$ shows undercutting requires a non-positive price and is therefore not worthwhile. Q.E.D.

APPENDIX 2. Proof of equilibrium existence for the integrative approach, $\phi \geq 1/3$.

Consider the point-profit function (of firm 0), $\pi_0(p_0, p^*; z) = p_0^{1-\alpha}\mathbb{P}_0(p_0, p^*; z)$, which is the profit earned at point z when firm 0 sets price p_0 and all other firms set equal prices, p^*, as given by (10). Following the method outlined at the beginning of Section 4, we can write, for $z \in [k\text{-} \ , (k+1)\text{-} \ ]$,

$$\mathbb{P}_0(p_0, p^*; z) = \exp\Delta\left\{\exp\Delta - 1 + \frac{\phi(1-\phi^m)}{1-\phi}\left[\phi^{-(k+1)} + \phi^k \exp\left[\frac{2tz}{\mu}\right]\right]\right\}^{-1}$$

where $\Delta \equiv [v(p_0) - v(p^*)]/\mu = [p^{*1-\alpha} - p_0^{1-\alpha}]/\mu(1-\alpha)$. Clearly $\Delta = 0$ corresponds to the symmetric candidate, and $\mathbb{P}_0(\cdot)$ then reduces to (11). Note that $\mathbb{P}_0(\cdot)$ is decreasing in z.

We can rewrite the point-profit function as $\Pi_0(\hat{v}_0, \tilde{v}^*; z) = \tilde{v}_0(1-\alpha)\mathbb{P}_0(\tilde{v}_0, \tilde{v}^*; z)$, where $\tilde{v}_0 \equiv \frac{p_0^{1-\alpha}}{1-\alpha}$ and $\tilde{v}^* \equiv \frac{p^{*1-\alpha}}{1-\alpha}$, so that it suffices to show that $\Pi_0 \equiv 2\int_0^{L/2}\Pi_0(\tilde{v}_0, \tilde{v}^*; z)dz$ is quasiconcave in v_0 given all other firms have "utility" levels $\tilde{v}^*$.

We now show that $\Pi_0(\tilde{v}_0, \tilde{v}^*; z)$ is quasiconcave in $\tilde{v}_0$, with maximum at $\tilde{v}_0^M$ and with a single inflection point at $\tilde{v}_0^I(z) > \tilde{v}_0^M(z)$, so that $\Pi_0(\cdot)$ is concave for $\tilde{v}_0 \leq \tilde{v}_0^I(z)$ and is decreasing for

$\tilde{v}_0 \geq \tilde{v}_0^I(z)$. First note that

$$\frac{\partial \Pi_0}{\partial \tilde{v}_0} = (1-\alpha)\mathbb{P}_0 + \tilde{v}_0(1-\alpha)\mathbb{P}_0(\mathbb{P}_0 - 1)/\mu$$

so that $\tilde{v}_0^M(z)$ is the unique solution to $\tilde{v}_0 = \mu/(1-\mathbb{P}_0)$ (the LHS is increasing in $\tilde{v}_0$ whereas the RHS is decreasing in $\tilde{v}_0$). Note that $\tilde{v}_0^M(z)$ is decreasing in z since $\mathbb{P}_0(\cdot)$ decreases with z. Likewise, the inflection point is the unique solution $\tilde{v}_0^I(z)$ to $\partial^2\Pi_0/\partial\tilde{v}_0^2 = 0$, or $\tilde{v}_0 = \mu/(\frac{1}{2} - \mathbb{P}_0)$, and again $\tilde{v}_0^I(z)$ decreases with z.

Hence if $\Pi_0(\tilde{v}_0, \tilde{v}^*; 0)$ is maximized at $\tilde{v}_0^M(0) \leq \tilde{v}_0^I(L/2)$ then $\Pi_0(\tilde{v}_0, \tilde{v}^*; z)$ is concave for all z for $\tilde{v}_0 \leq \tilde{v}_0^M(0)$ and $\Pi_0(\tilde{v}_0, \tilde{v}^*; z)$ is decreasing for all $\tilde{v}_0 \geq \tilde{v}_0^M(0)$ so that $\Pi_0(\tilde{v}_0, \tilde{v}^*)$ is concave for $\tilde{v}_0 \leq \tilde{v}_0^M(0)$ and decreasing for $\tilde{v}_0 \geq \tilde{v}_0^M(0)$ and is thus quasiconcave in $\tilde{v}_0$. Figure 3 illustrates. Note that if these conditions hold then $\tilde{v}_0^M(\frac{L}{2}) < \tilde{v}^* < \tilde{v}_0^M(0)$.

Insert Figure 3

We therefore need a sufficient condition for $\tilde{v}_0^M(0) \leq \tilde{v}_0^I(L/2)$. From the definitions of $\tilde{v}_0^M(0)$ and $\tilde{v}_0^I(L/2)$, it suffices that $\mu/(1-\mathbb{P}_0(\tilde{v}_0^M(0), \tilde{v}^*; 0) \leq \mu(\frac{1}{2} - \mathbb{P}_0(\tilde{v}_0^I(\frac{L}{2})), \tilde{v}^*; L/2))$, or $\mathbb{P}_0(\tilde{v}_0^M(0), \tilde{v}^*; 0) - \mathbb{P}_0(\tilde{v}_0^I(\frac{L}{2}), \tilde{v}^*; L/2) \leq 1/2$. Now, note that $\tilde{v}_0^M(0)$ must exceed $\tilde{v}^*$ even if the candidate equilibrium $\tilde{v}^*$ were in fact a profit minimum or a profit inflection point. This is because $\Pi_0(\tilde{v}_0, \tilde{v}^*)$ is strictly decreasing beyond $\tilde{v}_0^M(0)$ since Π_0 is falling for all z. It is therefore sufficient to prove that (i)

$$\frac{\mu}{1-\mathbb{P}_0(\tilde{v}^*, \tilde{v}^*; 0)} \leq \frac{\mu}{(1/2 - \mathbb{P}_0(\tilde{v}^*, \tilde{v}^*; L/2))}$$

and that (ii)

$$\frac{\partial}{\partial \tilde{v}_0}\left(\frac{\mu}{1/2-\mathbb{P}_0(\tilde{v}_0,\tilde{v}^*;L/2)}-\frac{\mu}{1-\mathbb{P}_0(\tilde{v}_0,\tilde{v}^*;0)}\right)\geq 0$$

for $\tilde{v}_0 \geq \tilde{v}^*$ (see Figure 4).

Insert Figure 4

To prove (i) it suffices to prove that $\mathbb{P}_0(\tilde{v}^*,\tilde{v}^*;0)-\mathbb{P}_0(\tilde{v}^*,\tilde{v}^*;L/2)\leq 1/2$; when that holds, it suffices to prove (ii) that $\frac{\partial}{\partial\tilde{v}_0}[\mathbb{P}_0(\tilde{v}_0,\tilde{v}^*;0)-\mathbb{P}_0(\tilde{v}_0,\tilde{v}^*;L/2)]\leq 0$ for $\tilde{v}_0\geq\tilde{v}^*$. Now, $\mathbb{P}_0(\tilde{v}^*,\tilde{v}^*;0)=(1-\phi)/(1-\phi^m)(1+\phi)$ and $\mathbb{P}_0(\tilde{v}^*,\tilde{v}^*;L/2)=(1-\phi)/(\phi^{-m}-1)(1+\phi)$, so that condition (i) becomes $\phi\geq 1/3$. To prove (ii) then holds, define $\theta_0\equiv\left(\frac{1-\phi^m}{1-\phi}\right)(1+\phi)-1$ and $\theta_{L/2}\equiv\frac{(\phi^{-m}-1)}{1-\phi}(1+\phi)-1$, so that $\mathbb{P}_0(\tilde{v}_0,\tilde{v}^*;0)-\mathbb{P}_0(\tilde{v}_0,\tilde{v}^*;L/2)\equiv G$, where

$$G=\exp\Delta\left\{\frac{\theta_{L/2}-\theta_0}{(\exp\ \tilde{\Delta}+\theta L/2)(\exp\ \Delta+\theta_0}\,)\right\}$$

and $\operatorname{sgn}\frac{dg}{d\tilde{v}_0}=\operatorname{sgn}\left[\theta_{L/2}\theta_0-\exp\ 2\Delta\right]$. We therefore wish to show that

$$1+\left(\frac{1+\phi}{1-\phi}\right)\left\{\phi^m-\phi^{-m}+\left(\frac{1+\phi}{1-\phi}\right)(\phi^m+\phi^{-m}-2)\right\}\geq\exp\ 2\tilde{\Delta}.$$

Since $\tilde{\Delta}\leq 0$ in the region under consideration, it suffices to show that the bracketed term is nonnegative. This is true because the term has the sign of $\phi^m+\phi^{1-m}-\phi-1$, which is convex for $\phi\in[0,1]$ with zero derivative at $\phi=1$ (where it is zero).